$C = (N + 1)(V/S\tau_N - 1)$ . For these skin thickness variations,

$$\frac{\Delta T_N}{\Delta T_0} = 2(S/S_0)^2 (\tau_0/\tau_N) \int_0^1 \frac{\xi d\xi}{1 + (N+1)[(V/V_0)(S_0/S)(\tau_0/\tau_N) - 1]\xi^N}$$
(2)

These results are shown in Fig. 1 for different values of the exponent, N, and for  $V=V_0$  and  $S=S_0$ . For each N a thickness ratio,  $\tau_n/\tau_0$ , exists for which  $\Delta T_N/\Delta T_0$  is a minimum. The linear variation, which would probably be the easiest to fabricate, gives as much as a 10.5% reduction in  $\Delta T_N$  with a factor of five change in skin thickness across the panel section half width and  $\tau_1 \approx \tau_0/3$ .

These results indicate that maximum temperatures in active cooled panels can be reduced with nonuniform skin thickness of the panel. A system weight reduction is implied because for an allowable temperature change across the panel sections either the skin volume can be reduced for a given passage spacing (about 10.5% for N=1), or the passage spacing can be increased for a given skin volume (about 3.5% for N=1). Whether these small savings will be achievable will depend, however, upon the structural considerations of the effects of variable skin thickness and fabrication costs.

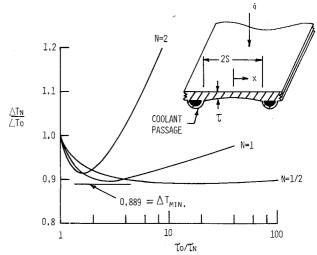


Fig. 1 The effect of nonuniform skin thickness on panel temperature change.

#### Reference

<sup>1</sup>Helenbrook, R. G. and Anthony, F. M., "Design of a Convective Cooling System for a Mach 6 Hypersonic Transport Airframe," CR-1918, Dec. 1971, NASA.

## **Technical Comments**

### Comment on "Film Reinforced Multifastened Mechanical Joints in Fibrous Composites"

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THE paper by Padawer<sup>1</sup> is concerned with a relatively new development in advanced composite materials, boron film. Being inplane isotropic instead of unidirectional as fibers are, it opens up possible structural advantages over filamentary composites.

While the boron/polyimide film has proved to be efficient in reinforcing basic laminates, there is not yet sufficient evidence to assess its potential as a hole reinforcement for practical aerospace construction. Accumulated evidence on bolted joints in advanced filamentary composites indicates that success with boron-film-reinforced bolted joints has yet to be demonstrated for conditions representative of current practice.

In order to explain how the work in the paper fits into the over-all picture, it is necessary to summarize the progress in advanced-composite bolted joints, in which certain characteristics have already been identified. First, ultimate static joint efficiencies are far below those developed by metal parts. The reason for this is that the composites are brittle while aluminum, titanium, and steel alloys exhibit considerable ductility in yielding locally to redistribute loads around a stress concentration. Second, the experimental joint strengths developed with mechanical fasteners in composites exceed significantly what would be expected according to a perfectly-elastic analysis. This gain in strength is due to the stress concentration relief afforded by even the limited (small) ductility exhibited by composites. For isotropic filament patterns (0°/45°/90°, -45°), this increase in strength ranges from a factor of 1.5 to one of about 3.0 for both HTS graphite and boron filaments in an epoxy matrix, the precise value depending on the joint geometry. Third, the stress concentration problem around bolt holes has been virtually completely eliminated when the longitudinal (0°) filaments in a strip of width 4 times the bolt diameter, in line with the bolt(s), have been replaced by S-glass filaments. The S-glass filaments have an ultimate elongation about 4 times as great as graphite and boron filaments. Since the average strain in the glass fibers is kept low by the adjacent advanced composite longitudinal fibers (outside the so-called softening strip), this relatively ductile region enables the gross laminate to be stressed far more highly than without such stress concentration relief. Indeed, in many such tests, failure has not been initiated at the bolt hole at all—the softening strips have been 100% effective. The bolt bearing stresses developed in composite laminates (both with

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and without softening strips) are quite high (150-250 ksi) for suitable laminate patterns, so integral local build ups within a laminate are capable of providing adequate strength without additional reinforcement in most cases.

Knowing how increasing the ultimate strain capacity in the vicinity of a bolt hole has dramatically increased the joint strengths, it is not clear how going in the opposite direction can improve matters further. The problem is that the boron film has an ultimate tensile strain of only about half that of the 0° filaments, so one would expect the boron film to break long before the filaments could be loaded up, so aggravating the stress concentration problem. Indeed, the author has recorded a reduction in relative displacement  $\delta$  prior to failure in his tests. The reason why an improvement in joint strength was demonstrated using boron film reinforcement is that the laminate selected (±45°) is used almost exclusively for shear and not direct loads, in which direction it is particularly weak. Likewise, the combination of boron films plus 0° fibers would be superior to 0° fibers alone in the vicinity of a bolt hole. A comparable improvement in structural patterns in the practical range (0°/±45°/90°) through (0°/ ±45°/0°) would be really significant but the limited testing in this range has not been encouraging.

The real problem with load sharing in multiple-fastener patterns is not in the line-abreast (parallel load path) case tested in the paper, but in the line-astern (series load path) configuration with the bolts distributed along the load direction. Only an exceedingly poor insert would weaken the strength of a line of bolts perpendicular to the load, but any insert with even a slight further limit on the ultimate strain capacity along the load direction will inevitably effect a drastic reduction of joint strength. All the load tends then to be picked up by the outer bolts, leaving the inner ones very lightly loaded when failure occurs at the outer bolts. Consequently it would be premature to infer that boron film in fibrous composites strengthens all joints of multibolt configuration which are of practical interest to aerospace construction.

One further factor of importance in the use and testing of the B/PI film is the greatly reduced perimeter available for bonding, per unit cross-sectional area, in comparison with boron filaments. The load transfer problems at the ends of brittle filaments are known to be severe and have demanded a level of attention far greater than needed for conventional (ductile) metal structures. Otherwise, premature failures not reflecting the true capabilities of the materials have occurred. If the use of this new film is to lead to structural efficiencies exceeding those of conventional metal structure, even greater care will be needed in the design details. For example, significant strength increases would be demonstrated by eliminating the out-of-plane eccentricity in the load path shown in Fig. 4 of the paper.

Further testing of the more critical practical joint configurations will be necessary before the B/PI film can be assessed for bolted joint strength improvement. That such tests are in progress is indicated in Padawer's conclusion. The results will be awaited with interest throughout the composites industry. One advantage of the B/PI film in reinforcing single bolt holes is that it occupies less space than an equivalent integral build up of the basic filamentary composite laminate. This feature is of importance when maintaining loft surfaces without introducing excessive eccentricities in load paths.

### Reference

<sup>1</sup>Padawer, G. E., "Film Reinforced Multifastened Mechanical Joints in Fibrous Composites," *Journal of Aircraft*, Vol. 10, No. 9, Sept. 1973, pp. 561-566.

# Comment on "A Compressibility Correction for Internal Flow Solutions"

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### Nomenclature

Q = mass flow

q =mass flow ratio (defined in text)

R = gas constant

r =stagnation density ratio (defined in text)

T = temperature

V = velocity

 $\gamma$  = ratio of specific heats

 $\rho = density$ 

Subscript

c = compressible

i = incompressible

t = stagnation

o =inlet or outlet plane

### Introduction

POTENTIAL flow solutions for the blade-to-blade velocity distribution in turbomachines are now an accepted part of current design procedure. Various methods are available, and some of these generate incompressible solutions quite rapidly. Compressible solutions, however, involve considerably greater running time on computers. Hence, a simple form of compressibility correction suitable for turbomachine blade passages is an attractive prospect. Lieblein and Stockman¹ propose such a correction, of an essentially empirical nature. The present Note offers three examples of the use of their method, to demonstrate the degree to which it can be generally applied.

### Analysis

It should be stated at the outset that a correction of this nature can only produce a compressible version corresponding to the same outlet gas angle as that of the original incompressible solution. This limitation can be a serious one, as experience has shown that for potential flow solutions to be correct, as judged by the usual criteria,\* the outlet angles for incompressible and compressible cases must often be different.

The correction of Ref. 1 for blade surface velocities is given in the following form

$$V_c/V_i = (\rho_i/\overline{\rho_c})^{V_i/\overline{V_i}} \tag{1}$$

where subscripts i and c relate to incompressible and compressible values respectively, and the bar refers to average flow conditions across the passage at the same axial station. For the application of Eq. (1) its authors assume that stagnation density  $(\rho_i)$  and mass flow are the same for both incompressible and compressible cases. Those restrictions are, however, unnecessary and it is possible to use this correction in more general circumstances. Consider two solutions

a) Incompressible

mass flow =  $Q_i$ ; density =  $\rho_i$ ; at any axial station mean passage velocity =  $\bar{V}_i$  and surface velocity =  $V_i$ 

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<sup>\*</sup> Either equal heights to the velocity peaks on suction and pressure surfaces immediately ahead of the trailing edge stagnation point, or, ignoring those peaks, extrapolations of the surface velocity curves which give zero trailing edge loading.